Verification and synthesis of dynamic systems with locks and variables

Corto Mascle
joint work with Anca Muscholl, Igor Walukiewicz

LaBRI, Bordeaux

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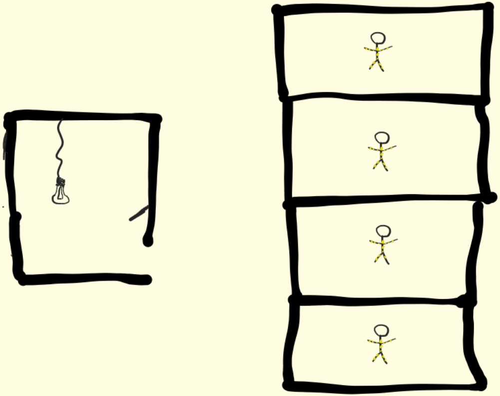
Synthesis Days

The prisoners and the lightbulb



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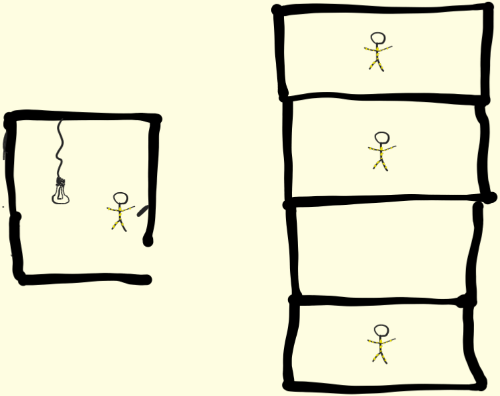
- Everyday, one is picked at random and taken to a room with a lightbulb and a switch, then brought back to the cell.



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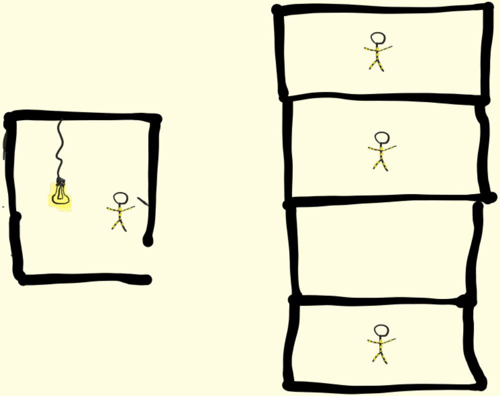

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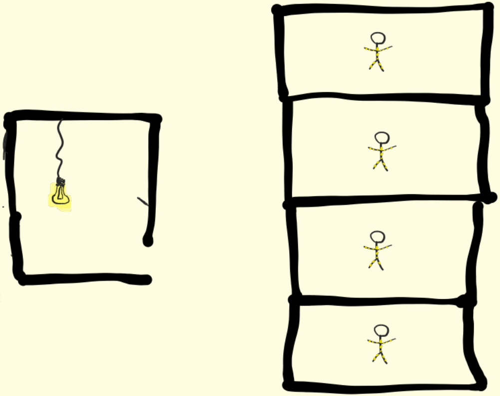
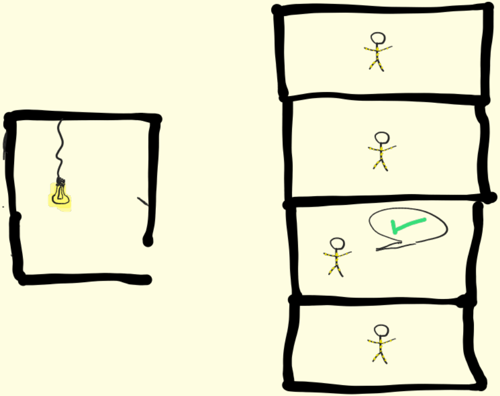


The prisoners and the lightbulb

- 4 prisoners are waiting in their cells
- Everyday, one is picked at random and taken to a room with a lightbulb and a switch, then brought back to the cell.
- At any point a prisoner can claim that all prisoners have been in the cell at least once.
  They win if it is true, otherwise they lose.



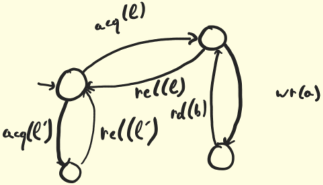
Formal problem

We have:

- A finite set of processes
- A finite set of variables
- A finite set of locks

Each process is a finite-state transition system with operations

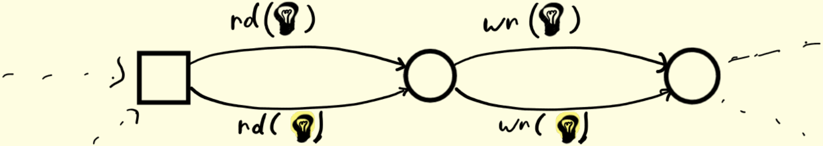
$$\begin{cases} wr(a) & a \in \Sigma \\ rd(a) & \\ acq(\ell) & \ell \in Locks \\ rel(\ell) & \end{cases}$$



```
def f(x,ℓ):
    acq(ℓ)
    if (x = a):
        x ← b
    rel(ℓ)
```

Formal problem

Processes have controllable ○ and uncontrollable □ states.



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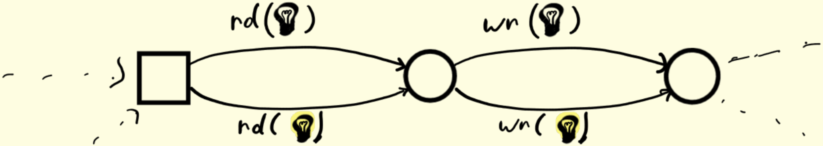

A **strategy** σ_p for process p is a function choosing the next action of p from controllable states with as input the local run seen so far.

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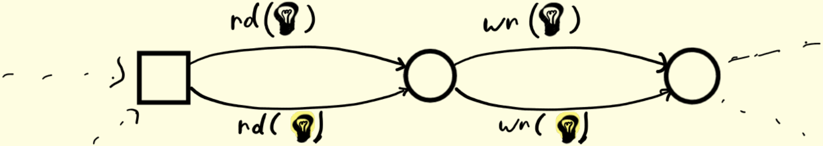

A **strategy** σ_p for process p is a function choosing the next action of p from controllable states with as input the local run seen so far.

Specifications = boolean combinations of local regular conditions.

- Processes have controllable and uncontrollable states
- Strategies are local, ie, only use the sequence of local actions of the process as input.

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- Strategies are local, ie, only use the sequence of local actions of the process as input.

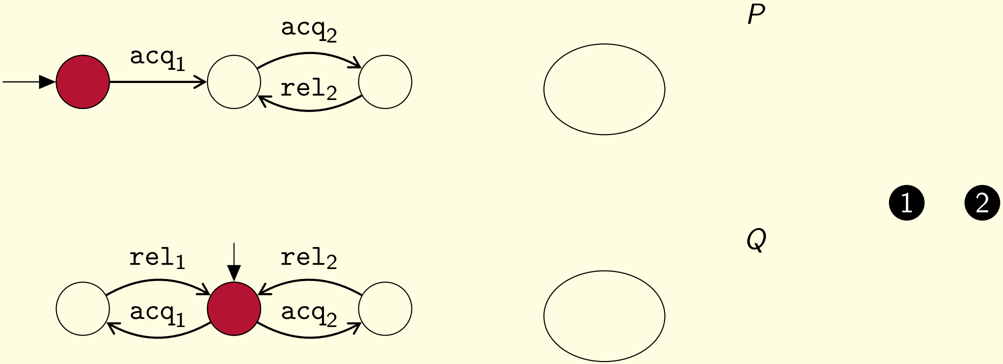
Problem

Is there a strategy $\sigma = (\sigma_p)_{p \in Proc}$ ensuring that there is no execution accepted by $\mathcal{A}$?

PART I

Verification

Lock-sharing systems[1]



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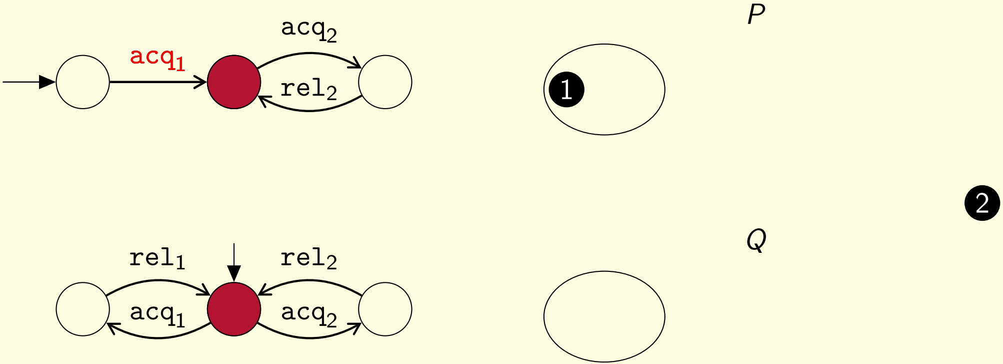
[1] Kahlon, Ivancic, Gupta CAV 2005

Lock-sharing systems[1]



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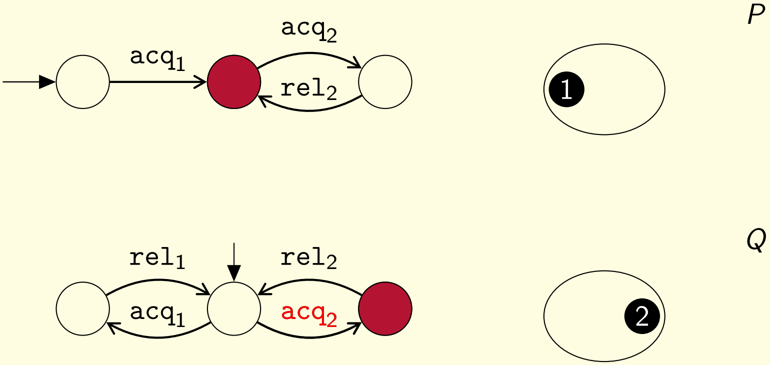

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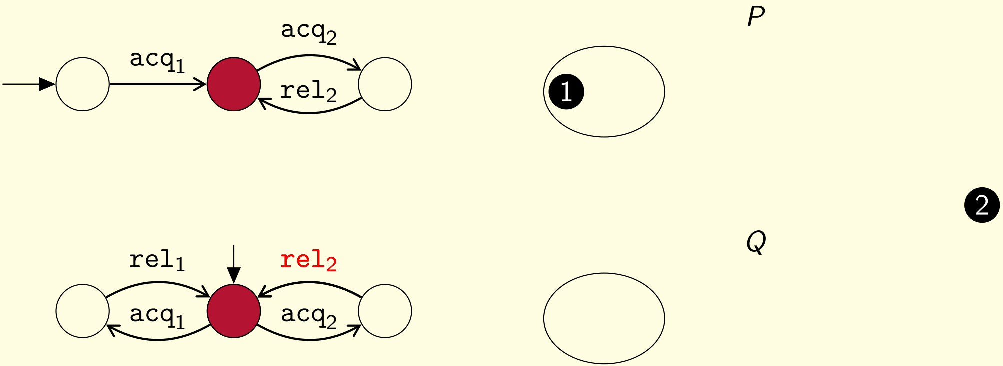

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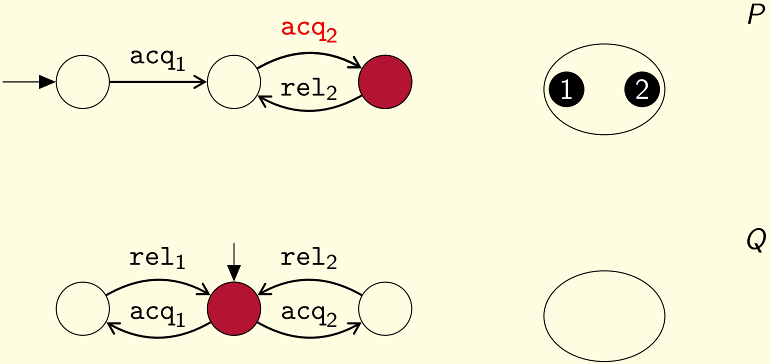

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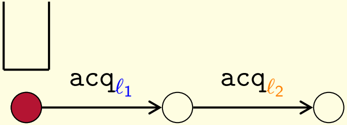
Restriction: Nested locking

All processes acquire and release locks in a **stack-like order**, i.e., a process can only release the lock it acquired the latest.



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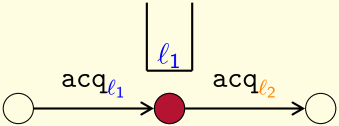

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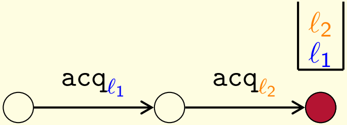

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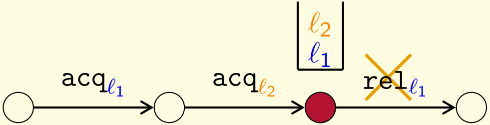

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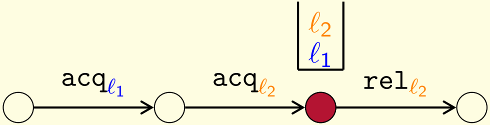

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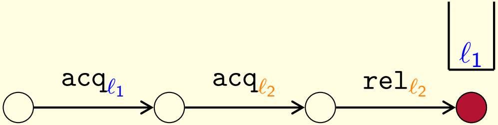

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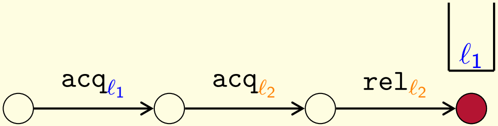

⚠ We assume nested locking in the rest of the presentation.

This restricts communication between processes.

Dynamic LSS

- We want to allow an unbounded number of processes and locks.

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- A process can spawn other processes

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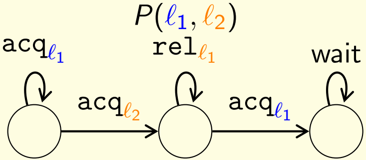
- A process can spawn other processes

- A process takes parameters, represented by *lock variables*

$Proc = \{P(\ell_1, \ell_2), Q(\ell_1, \ell_2, \ell_3), R(), \ldots\}$

Dynamic LSS[2]

Locks : 🟦 🟧



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Dynamic LSS[2]

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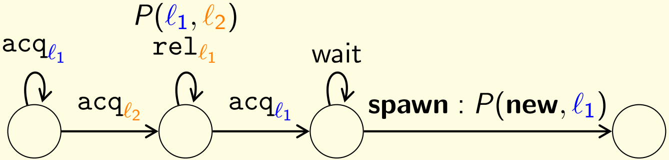

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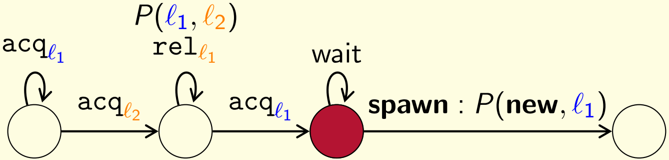

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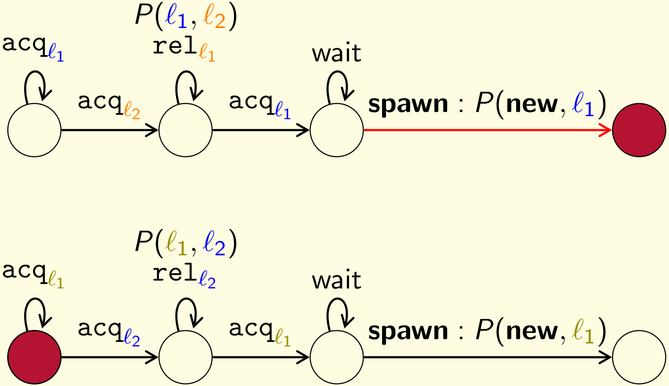

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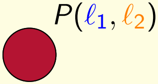
Tree representation



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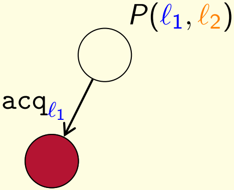

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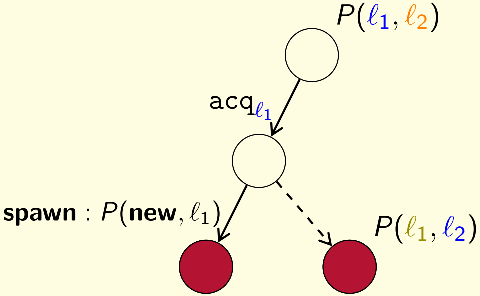

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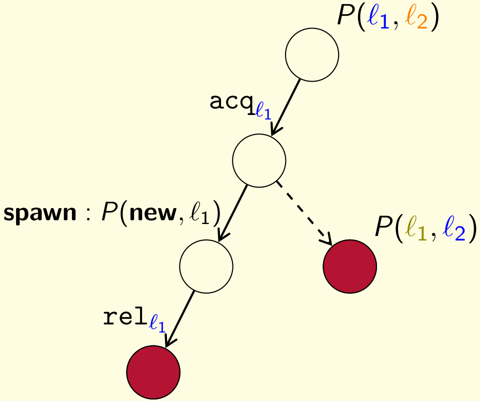

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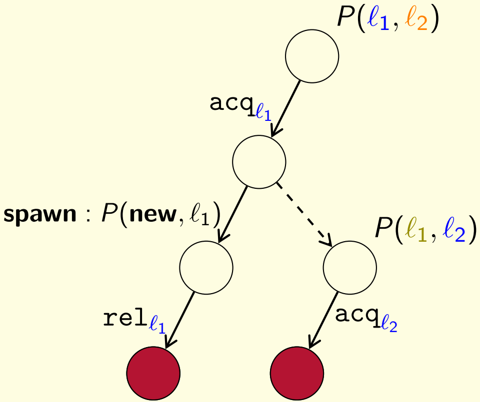

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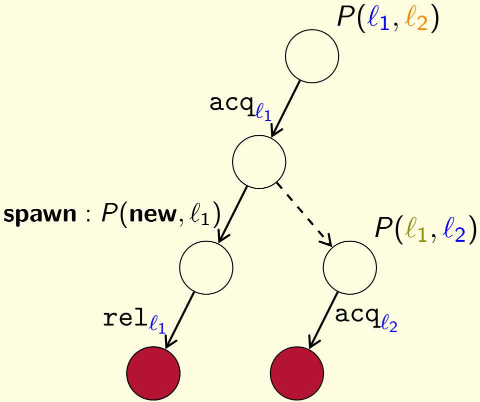
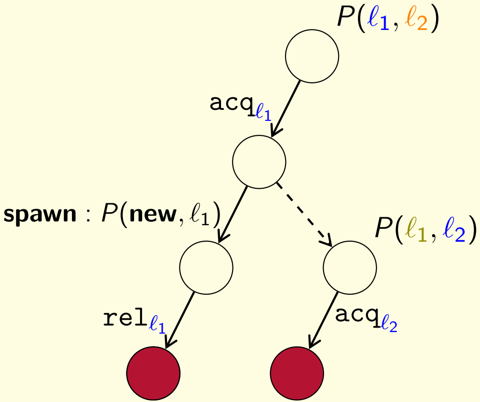


Specifications are ω-regular tree languages.

Tree representation



Specifications are ω-regular tree languages.

"Every process is blocked after some point"

"Finitely many processes are spawned"

"Infinitely many processes reach an error state q_{err}"

Deadlocks

Regular model-checking problem

Input: A DLSS $\mathcal{D}$ and a parity tree automaton $\mathcal{A}$.

Output: Is there a run of $\mathcal{D}$ accepted by $\mathcal{A}$?

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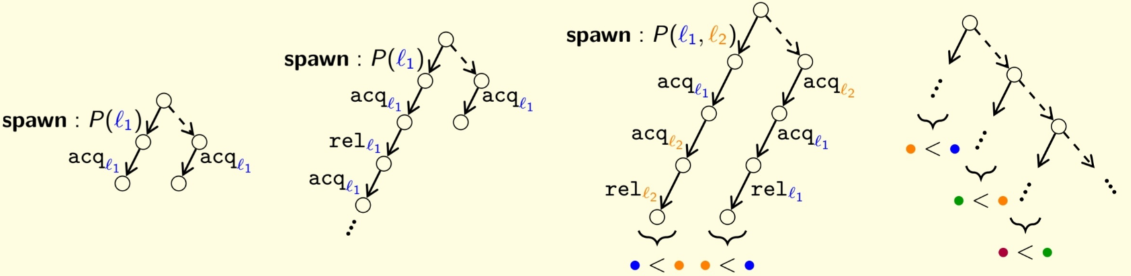
Problem: characterise trees that represent actual executions.



Lemma

The set of execution trees of a DLSS is recognised by a Büchi tree automaton of exponential size.

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$P(\ell_1, \ell_2, \ell_3, \ell_4)$

For each node we guess a label of the form

- "ℓ_1 is taken and will never be released", "ℓ_2 will be acquired infinitely many times", ...
- $$\ell_3 \begin{array}{l} < \ell_1 \\ < \ell_4 \end{array}$$

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The automaton checks that:

- the labels are consistent
- There exists a well-founded linear ordering on locks in which all local orders embed. (Technical part, also see related work [Demri Quaas, Concur '23])

Theorem [M., Muscholl, Walukiewicz Concur 2023]

Regular model-checking of DLSS is EXPTIME-complete, and
PTIME for fixed number of locks per process and parity index.

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PTIME for fixed number of locks per process and parity index.

What about pushdown processes?

Right-resetting pushdown tree automata

Right-resetting = the stack is emptied every time we go to a right child.

Lemma

Emptiness is decidable in PTIME for right-resetting parity pushdown tree automata when the parity index is fixed.

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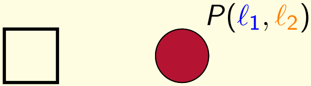
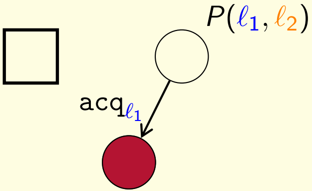
What about shared variables?

DLSS with variables



We add a register and operations `wr` and `rd` writing and reading letters from a finite alphabet in the register.

DLSS with variables



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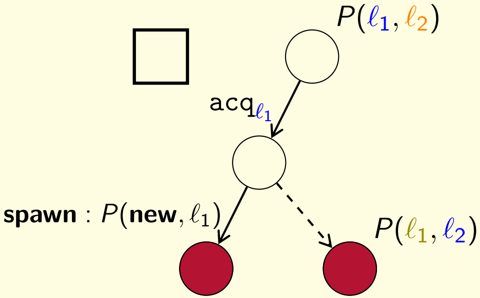

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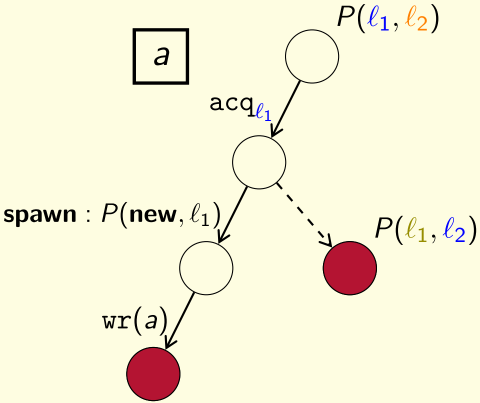

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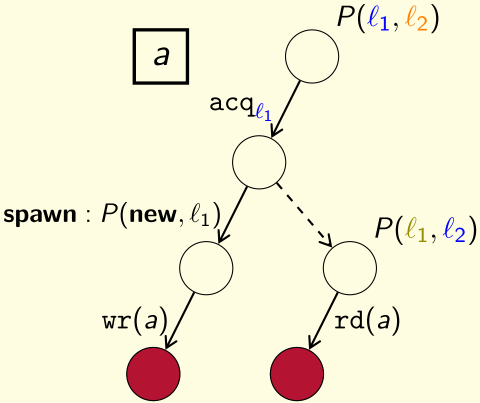

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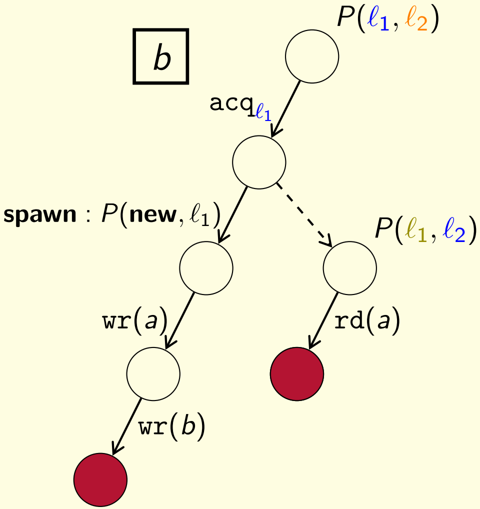

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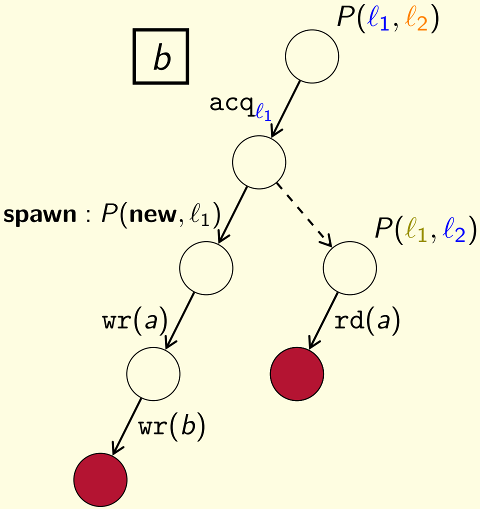

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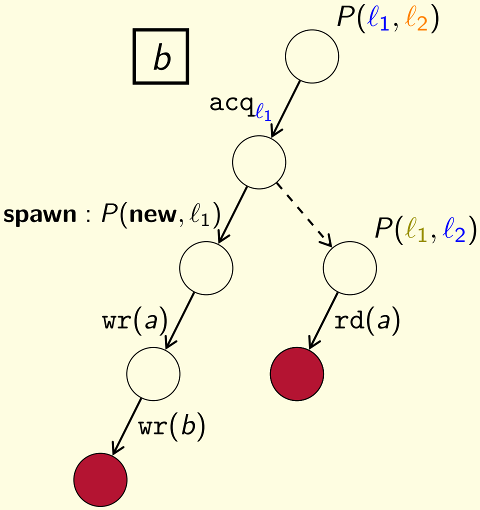
Sets of runs are no longer regular.

DLSS with variables



We add a register and operations `wr` and `rd` writing and reading letters from a finite alphabet in the register.

Sets of runs are no longer regular.

Theorem

State reachability is undecidable for DLSS with variables.

Bounded writer reversals

Writer reversal = the process writing in the shared register changes.

---

[3] Atig, Bouajjani, Kumar, Saivasan FSTTCS 2014

Bounded writer reversals

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State reachability is decidable for DLSSV with bounded writer reversals.

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Bounded writer reversals

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Theorem

State reachability is decidable for DLSSV with bounded writer reversals.

It is undecidable when the processes are pushdown systems[3].

---

[3] Atig, Bouajjani, Kumar, Saivasan FSTTCS 2014

Proof sketch

Consider a run with one process writing and others reading.

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Phase: run section where

- the writer is in the same state and has the same locks at the start and at the end,
- none of the locks used by the writers in the phase are held by another process at the start or the end

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- none of the locks used by the writers in the phase are held by another process at the start or the end

Lemma

Every finite run with a single writer can be cut into $2^{O(|Q|)}$ phases.

Proof sketch

Consider one phase.

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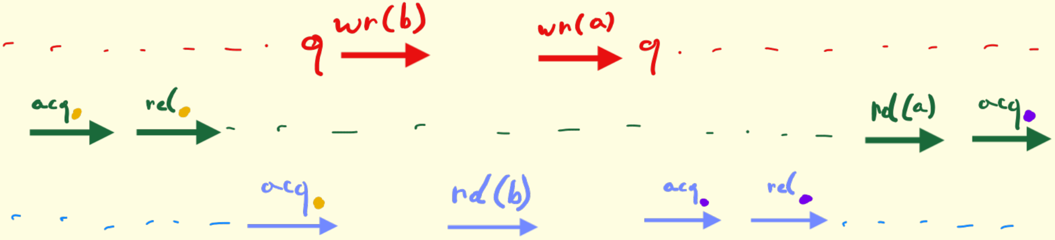
Lemma

Every phase can be replaced by a sequence of phases where at most one reader moves.



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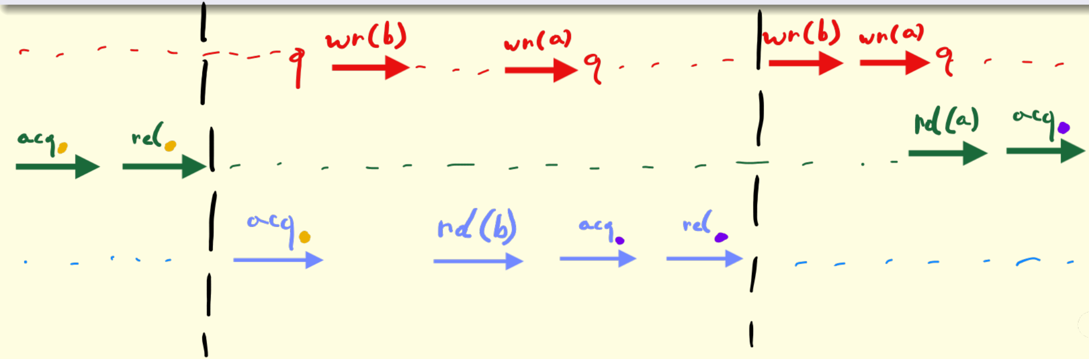

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Construct $\mathcal{A}$ that:

- guesses a partition of the tree in $K2^{O(|Q|)}$ phases, each with a single writer.
- checks lock conditions
- checks compatibility of each reader with the writer

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To sum up

	State reach	ω-regular
Nested LSS	✓	✓
Nested Dynamic LSS	✓	✓
Nested DLSS + var with bounded writer switches	✓	?
Nested DLSS + var	✗	✗

PART II

Controller synthesis

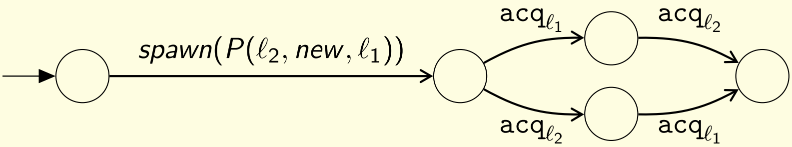
- Processes have controllable and uncontrollable states
- Strategies are local, ie, only use the sequence of local actions of the process as input.
- Every copy of each process uses the same strategy

Problem

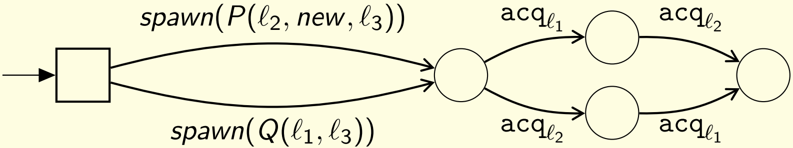
Is there a strategy $\sigma = (\sigma_p)_{p \in Proc}$ ensuring that there is no execution accepted by $\mathcal{A}$?

Example

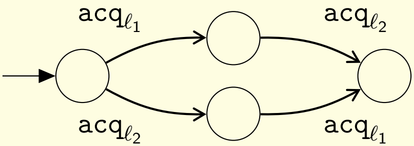
$P_{init}(\ell_1, \ell_2)$:



$P(\ell_1, \ell_2, \ell_3)$:



$Q(\ell_1, \ell_2)$:



With locks only (DLSS)

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Strategy $(\sigma_p)_{p \in Proc} \rightarrow$ set of local runs
Is there a strategy such that we cannot form a tree accepted by $\mathcal{T}$ whose left branches are those local runs?

With locks only (DLSS)

We define *types* of local runs (= left branches).
The behaviour of a strategy $\sigma = (\sigma_p)_{p \in Proc}$ is the set of types of left branches compatible with it.

Lemma

Whether $(\sigma_p)_{p \in Proc}$ is winning only depends on its behaviour.

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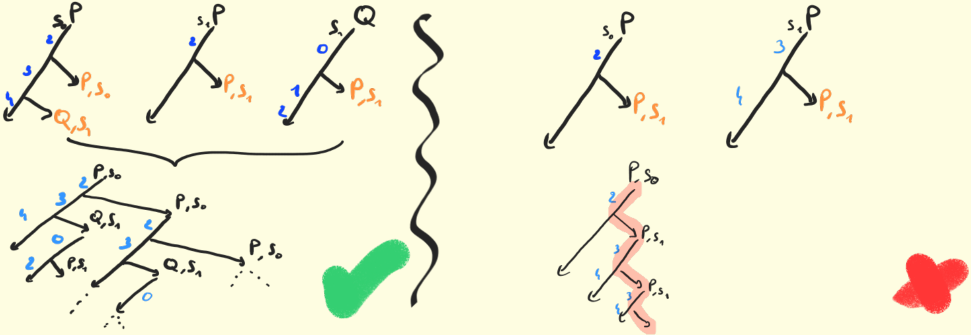

With locks only (DLSS)

Theorem

The controller synthesis problem is decidable over DLSS.

With locks only (DLSS)

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The controller synthesis problem is decidable over DLSS.

Algorithm:

- Enumerate sets of profiles
- For each one, test whether there is a strategy yielding that set of profiles
- If there is one, $\underbrace{\text{there is one with bounded memory:}}_{\text{From } \omega\text{-regular games}}$ check whether it is winning.

With locks and variables

With variables, none of this works!

- Sets of execution trees are not regular
- “Pumping argument” used for verification does not extend to adversarial setting.

What is left to do

Conjecture

Verification of nested DLSS with variables and bounded writer reversals against ω-regular tree specifications is decidable.

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Controller synthesis of nested DLSS with variables and bounded writer reversals against ω-regular tree specifications is decidable.

Formal problem

Problem

Given a system with processes, locks and variables and a specification φ, can we find a family of strategies $(\sigma_p)_{p \in \texttt{Proc}}$ guaranteeing φ?

Problem 1

I: A system $\mathcal{S}$, $K \in \mathbb{N}$ and a specification φ

O: Is there a family of strategies $(\sigma_p)_{p \in \texttt{Proc}}$ guaranteeing φ over runs with $\leq K$ writer reversals?

Problem 2

I: A system $\mathcal{S}$, $K \in \mathbb{N}$ and a specification φ

O: Is there a family of strategies $(\sigma_p)_{p \in \texttt{Proc}}$ guaranteeing φ and that all runs have $\leq K$ writer reversals?

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Algorithm:

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For $K = 0$ to $+\infty$ do
    If $\exists(\sigma_p)$ such that $\phi \wedge \leq K$      $\rightarrow$ return YES
    If $\nexists(\sigma_p)$ such that $\leq K \Rightarrow \phi$   $\rightarrow$ return NO
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Other directions

- General approach to local controller synthesis
- Parameterised complexity
- Strategies using more information

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Thanks!